Resonant Non-Gaussianity

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arXiv:1002.0833

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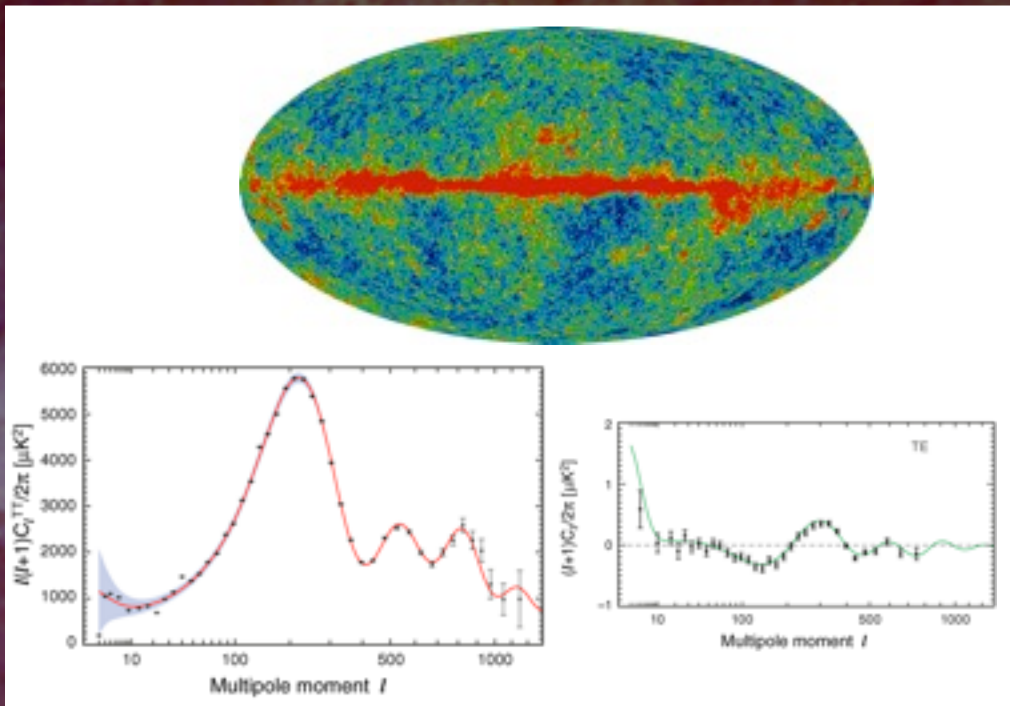
University of Michigan, May 14, 2010

Image: ESA/Planck

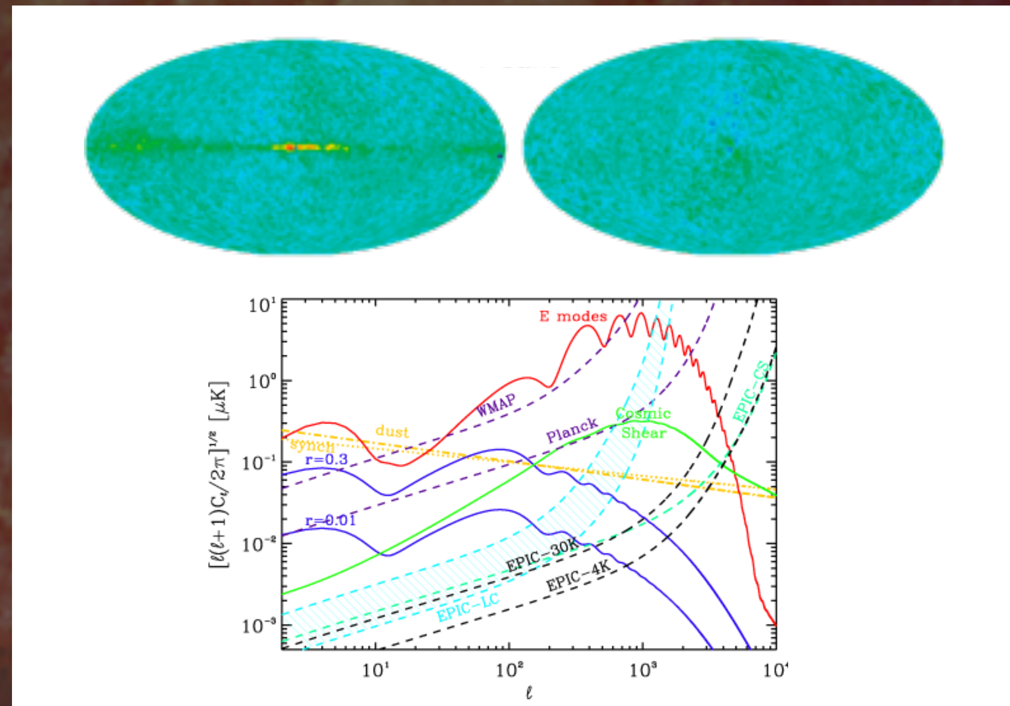
Motivation

(Jarosik et al. 2010)



(Larson et al. 2010)

(Jarosik et al. 2010)



(Bock et al. 2009)

If a tensor signal is seen, the inflaton must have moved over a super-Planckian distance in field space* **(Lyth 1996)**

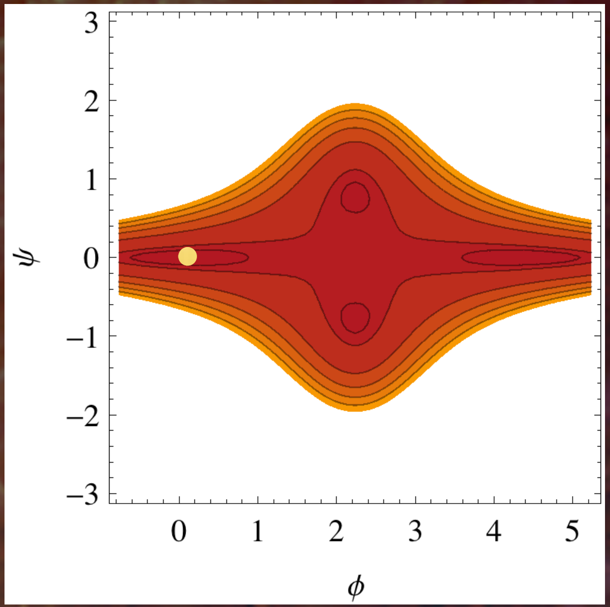
* For single field models with canonical kinetic term

Motivation

This is hard to control in an EFT field theory

$$V(\phi) = V_0 + \frac{1}{2}m^2\phi^2 + \frac{1}{3}\mu\phi^3 + \frac{1}{4}\lambda\phi^4 + \phi^4 \sum_{n=1}^{\infty} c_n \left(\phi/\Lambda\right)^n$$

$$(\Lambda < M_p)$$



The c_n are typically unknown.
Even if they were known, the effective theory is generically expected to break down for $\phi > \Lambda$, e.g. because other degrees of freedom become light.

Motivation

Possible Solution:

Use a field with a shift symmetry, e.g. axion.
Break the shift symmetry in a controlled way.

first example in string theory

Silverstein, Westphal, arXiv:0803.3085

(see Marco Peloso's and Enrico Pajer's talks for further references)

If the inflaton is an axion, periodic contributions to the potential can arise leading to

$$V(\phi) = V_0(\phi) + \Lambda^4 \cos\left(\frac{\phi}{f}\right)$$

numerical studies

Chen, Easther, Lim, arXiv:0801.3295

Hannestad, Haugboelle, Jarnhus, Sloth, arXiv:0912.3527

Summary of Results

The primordial power spectrum

The usual slow-roll derivation breaks down because of parametric resonance, and the Mukhanov-Sasaki equation has to be solved.

$$\frac{d^2\mathcal{R}_k}{dx^2} - \frac{2(1+2\epsilon+\delta)}{x}\frac{d\mathcal{R}_k}{dx} + \mathcal{R}_k = 0$$

with
$$\epsilon = \epsilon_* - 3bf\sqrt{2\epsilon_*}\cos\left(\frac{\phi_k + \sqrt{2\epsilon_*}\ln x}{f}\right)$$

$$\delta = \delta_* - 3b\sin\left(\frac{\phi_k + \sqrt{2\epsilon_*}\ln x}{f}\right)$$

$x = k/aH$ and $b = \Lambda^4/V'(\phi_*)f$

Summary of Results

The primordial power spectrum

$$\frac{d^2\mathcal{R}_k}{dx^2} - \frac{2(1+\delta_{\rm osc}(x))}{x}\frac{d\mathcal{R}_k}{dx} + \mathcal{R}_k = 0$$

Look for a solution

$$\mathcal{R}_k(x) = \mathcal{R}_{k,0}^{(o)}\left[i\sqrt{\frac{\pi}{2}}x^{3/2}H_{3/2}^{(1)}(x) - c_k^{(-)}(x)i\sqrt{\frac{\pi}{2}}x^{3/2}H_{3/2}^{(2)}(x)\right]$$

Then for large x

$$\frac{d}{dx}\left[e^{-2ix}\frac{d}{dx}c_k^{(-)}(x)\right] = -2i\frac{\delta_{\rm osc}(x)}{x}$$

Summary of Results

The primordial power spectrum

(Linear potential with $f = 10^{-3} M_p$, $b = 10^{-2}$.)

Summary of Results

The primordial power spectrum

One finds

$$\Delta^2_{\mathcal{R}}(k) = \Delta^2_{\mathcal{R}}(k_*) \left(\frac{k}{k_*}\right)^{n_s - 1} \left[1 + \delta n_s \cos\left(\frac{\phi_k}{f}\right)\right]$$

with

$$n_s = 1 - 4\epsilon_* - 2\delta_* \qquad \text{and} \qquad \delta n_s = 3b\left(\frac{2\pi f}{\sqrt{2\epsilon_*}}\right)^{1/2}$$

(This assumes $\frac{f}{\sqrt{2\epsilon_*}} \ll 1$. For the general case see our paper.)

For constraints on these parameters from WMAP5 for a linear potential, see

Flauger, McAllister, Pajer, Westphal, Xu, arXiv:0907.2916

Summary of Results

The bispectrum

Models with large $\dot{\delta}$ can lead to large non-Gaussianities

Chen, Easther, Lim, arXiv:0801.3295

$$\langle \mathcal{R}(\mathbf{k_1},t)\mathcal{R}(\mathbf{k_2},t)\mathcal{R}(\mathbf{k_3},t)\rangle =^*$$

$$-i\int_{-\infty}^{t} dt' \langle [\mathcal{R}(\mathbf{k_1},t)\mathcal{R}(\mathbf{k_2},t)\mathcal{R}(\mathbf{k_3},t), H_I(t')]\rangle$$

with

$$H_I(t) \supset -\int d^3x \; a^3(t)\epsilon(t)\dot{\delta}(t)\mathcal{R}^2(\mathbf{x},t)\dot{\mathcal{R}}(\mathbf{x},t)$$

* with slight abuse of notation

Summary of Results

The bispectrum

After some algebra

$$\frac{\mathcal{G}(k_1,k_2,k_3)}{k_1k_2k_3} = \frac{1}{8}\int_0^\infty dX\,\frac{\dot{\delta}}{H}e^{-iX}$$

$$\left[-i - \frac{1}{X}\sum_{i\neq j}\frac{k_i}{k_j} + \frac{i}{X^2}\frac{K(k_1^2+k_2^2+k_3^2)}{k_1k_2k_3}\right] + c.c$$

$$K = k_1 + k_2 + k_3$$

Summary of Results

The bispectrum

$$\frac{\mathcal{G}(k_1,k_2,k_3)}{k_1k_2k_3} = f^{\rm res}\left[\sin\left(\frac{\sqrt{2\epsilon_*}}{f}\ln K/k_*\right)\right. \\ \left. + \frac{f}{\sqrt{2\epsilon_*}}\sum_{i\neq j}\frac{k_i}{k_j}\cos\left(\frac{\sqrt{2\epsilon_*}}{f}\ln K/k_*\right) + \ldots\right]$$

with

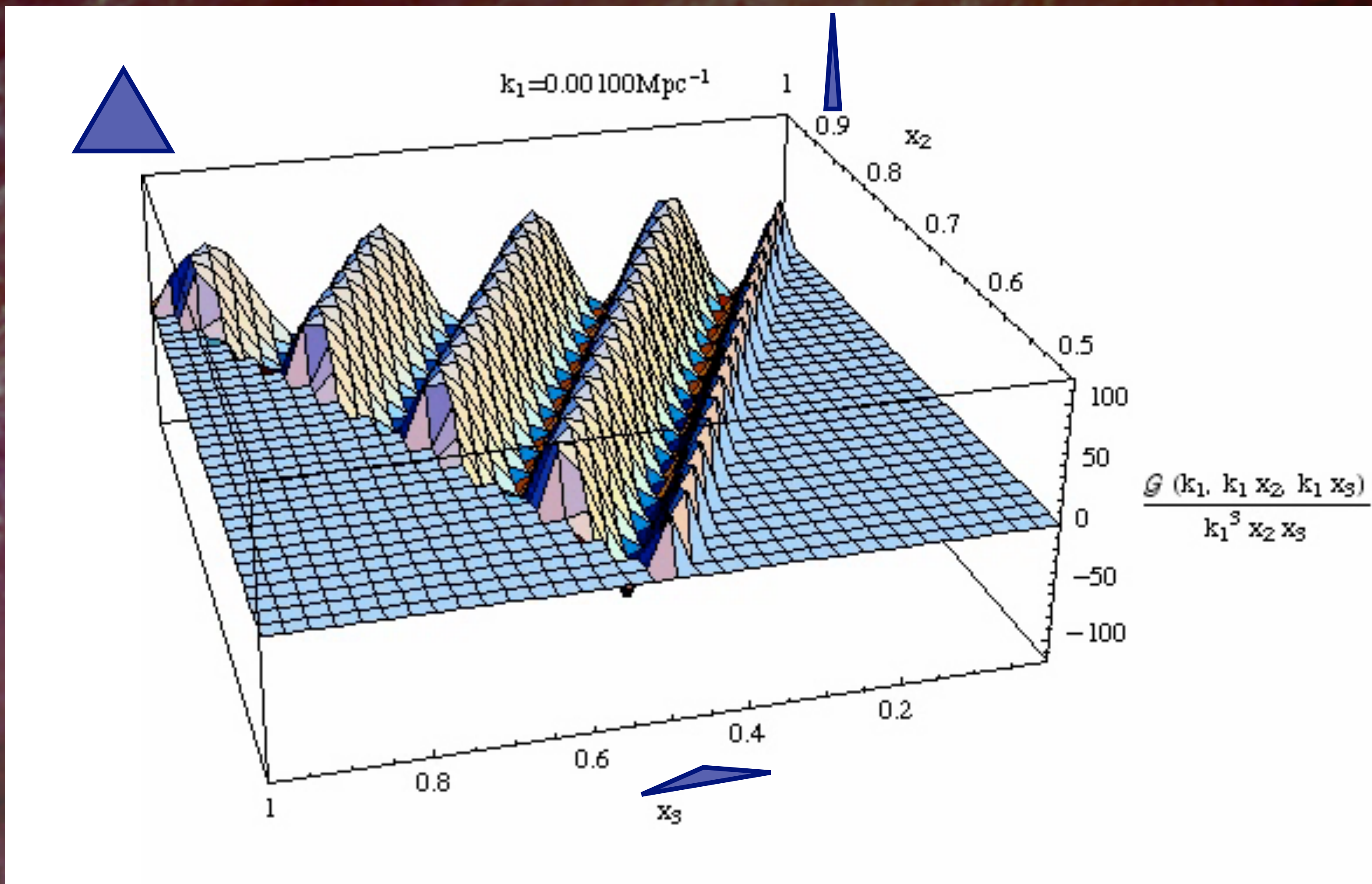
$$K = k_1 + k_2 + k_3$$

$$f^{\rm res} = \frac{3b_*\sqrt{2\pi}}{8}\left(\frac{\sqrt{2\epsilon_*}}{f}\right)^{3/2}$$

This satisfies the consistency condition.

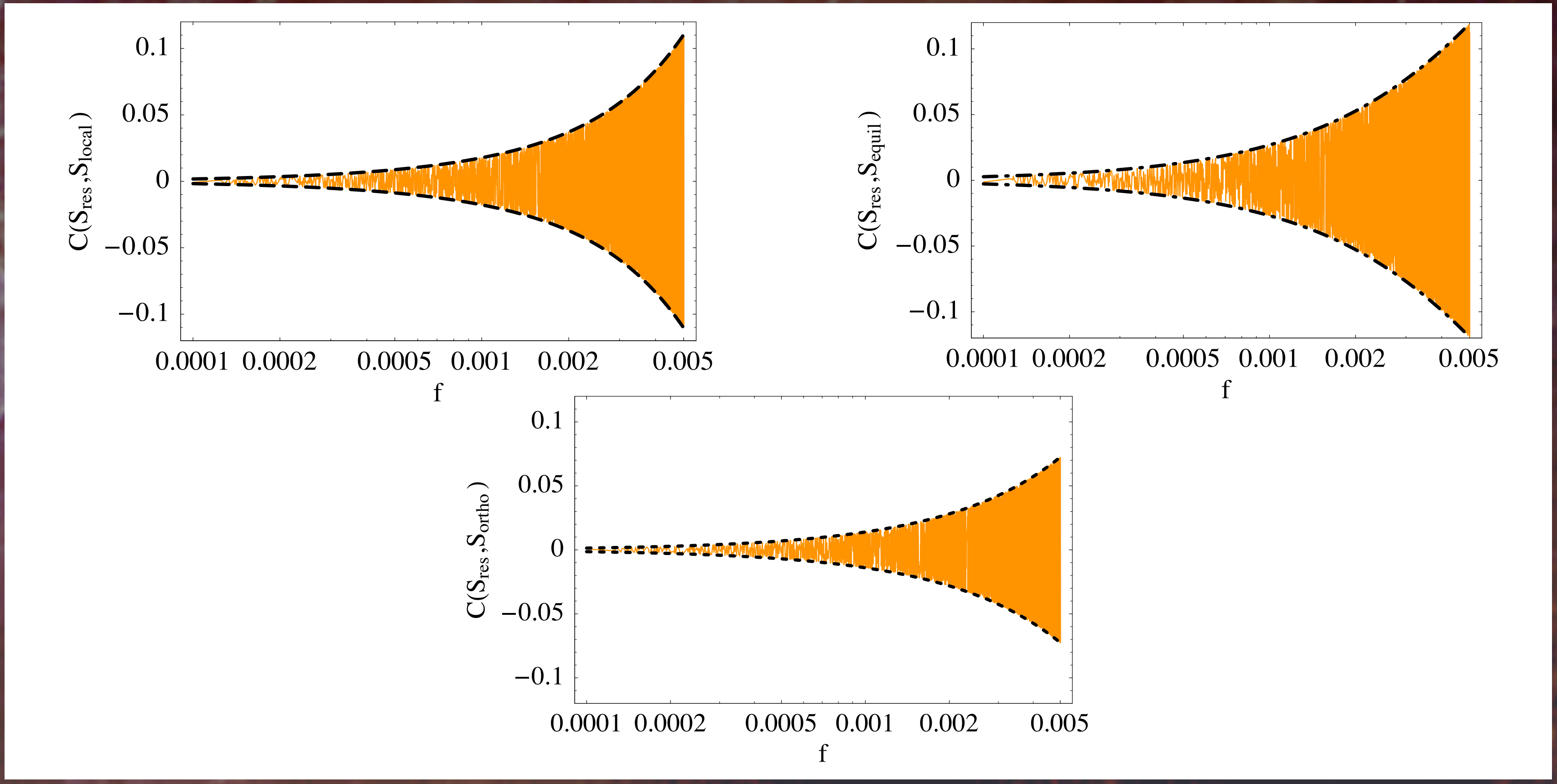
Summary of Results

The bispectrum



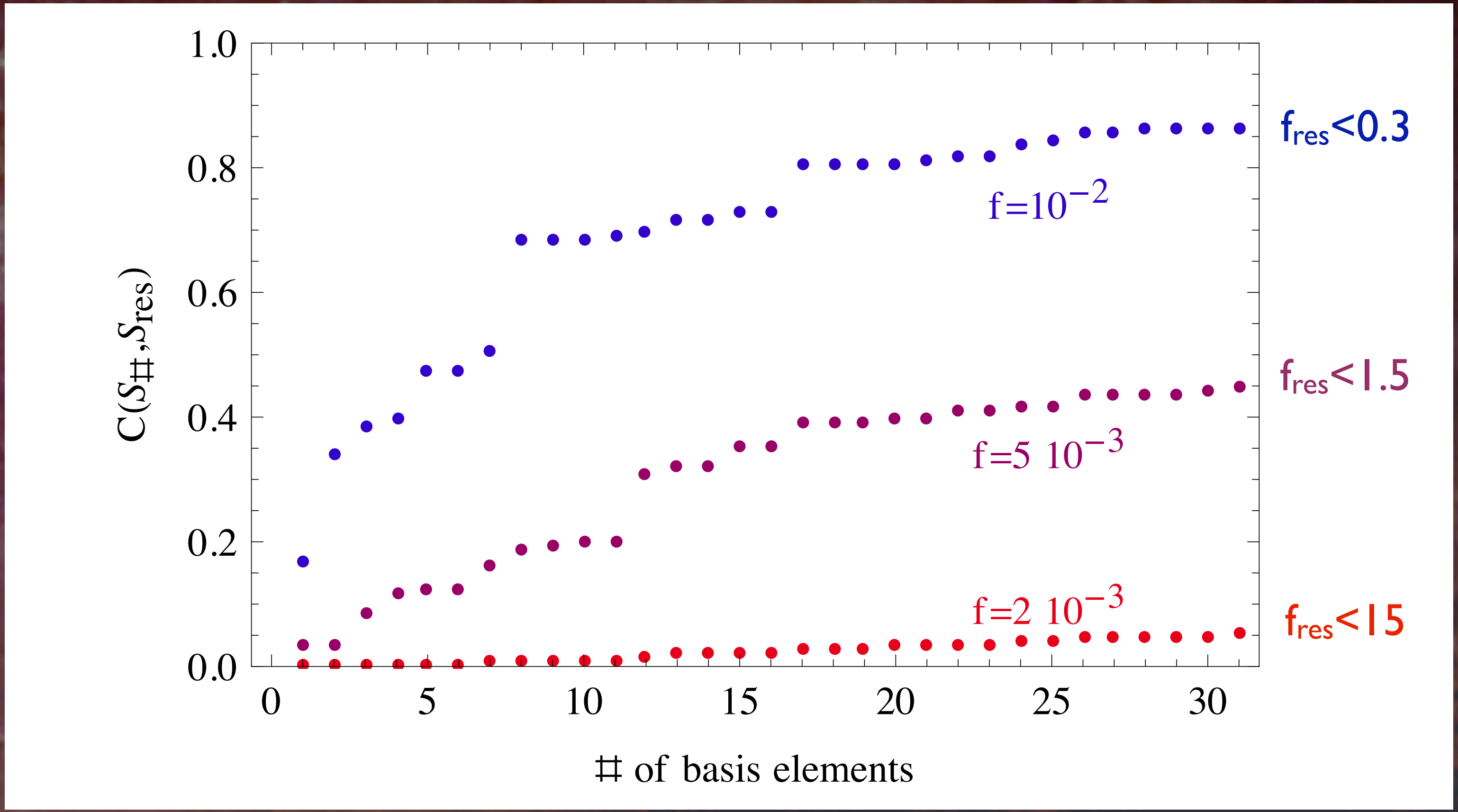
Summary of Results

Existing constraints on local, equilateral, and orthogonal shapes cannot be used to infer constraints on this shape.



Summary of Results

An expansion in a factorizable polynomial basis is limited to larger axion decay constants for which the amplitude is small.



Conclusions

- This shape of non-Gaussianities might be present in the case of large field inflation, but is currently essentially unconstrained.
- Techniques to measure general shapes including this shape are desirable

  Fergusson, Liguori, Shellard, arXiv:0912.5516, ...

  Meerburg, arXiv:1006.2771

- Maybe our analytic results will aid in constraining this shape...
- ...but there is still more work to be done to ensure we do not miss out on interesting physics


Thank you